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A characterization of PSL(2, 11) and S_5

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The symmetric group S_5 of degree five and the two dimensional projective special linear group PSL(2, 11) over the field of eleven elements are doubly transitive permutation groups of degree five and eleven, respectively, in which the stabilizer of two points is isomorphic to the symmetric group S_3 of degree three.

Let Ω be the set of points 1, 2, $\cdots$, n, where n is odd. Let G be a doubly transitive permutation group in which the stabilizer $G_{1,2}$ of the points 1 and 2 has even order and a Sylow 2-subgroup K of $G_{1,2}$ is cyclic. In the case $G_{1,2}$ is cyclic, Kantor-O'Nan-Seitz and the auther proved independently that G contains a regular normal subgroup ([4] and [8]). In this lecture we shall study the case $G_{1,2}$ is not cyclic . Let τ be the unique involution in K. By a theoem of Witt ([10]) the centralizer $C_G(\tau)$ of τ in G acts doubly transitively on the set $I(\tau)$ consisting of points in Ω fixed by τ.

The purpose of this lecture is to prove the following theorem.

Theorem. Let G, $G_{1,2}$, τ and $I(\tau)$ be above. Assume that all Sylow subgroups of $G_{1,2}$ are cyclic, the image of the doubly transitive permutation representation of $C_G(\tau)$ on $I(\tau)$ contains a regular normal subgroup and that G does not contain a regular normal subgroup. If G has two classes of involutions, then G is isomorphic to S_5 and $n = 5$. If G has one class of involutions and τ is not contained in the center of $G_{1,2}$, then G is isomorphic to $PSL(2, 11)$ and $n = 11$.

In [7] we proved this theorem in the case that the order of $G_{1,2}$ equals $2p$ for an odd prime number p.

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